Semester 1 Examination 2012

Question/Answer Booklet

MATHEMATICS 3CD

Section One:
Calculator-free

Name of Student: _____

Name of Teacher(Circle) Kirou Menzies Hoy Orton Sanders

Time allowed for this section
Reading time before commencing work: 5 minutes
Working time for this section: 50 minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the student
Standard items: pens, pencils, pencil sharpener, eraser, correction fluid/tape, ruler, highlighters
Special items: nil

Important note to students
No other items may be used in this section of the examination. It is your responsibility to ensure that you do not have any unauthorised notes or other items in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	6	6	50	50	
Section Two Calculator-assumed	12	12	100	100	
			Total	150	100

Instructions to students

1 Write your answers in the spaces provided in this Question/Answer Booklet. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer. If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued. i.e. give the page number. Fill in the number of the question(s) that you are continuing to answer at the top of the page.

2 **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

3 It is recommended that you **do not use pencil**, except in diagrams.

QUESTION	MARKS AVAILABLE	MARKS GAINED
1	8	
2	9	
3	7	
4	8	
5	10	
6	8	
TOTAL	50	

Section One: Calculator-free **(50 marks)**

This section has **six (6)** questions. Answer all questions. Write your answers in the spaces provided.

Working time: 50 minutes

Question 1 **(8 marks)**

(a) Simplify

(i) $\frac{x^2+5x-14}{5x^2-20} \div \frac{x^2+12x+35}{4x+20}$ (ii) $\frac{1}{x^2+x} + \frac{2}{x^2+2x}$ (4)

(b) The functions $f(x)$ and $g(x)$ are defined as follows

$f(x) = x^2 - 4$ and $g(x) = \sqrt{x-5}$

(i) Determine expressions for $f[g(x)]$ and $g[f(x)]$. (2)

(ii) Determine the range of $f[g(x)]$. (1)

(iii) Determine the domain of $g[f(x)]$. (1)

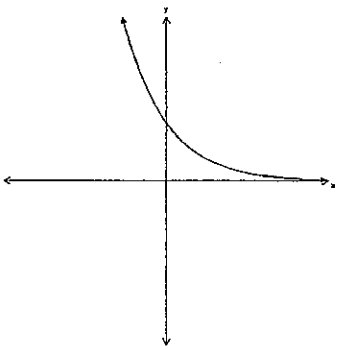
Question 2 **(9 marks)**

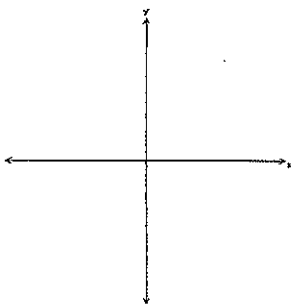
(a) Differentiate the following with respect to x.

(i) $f(x)=\dfrac{-x}{x^2+1}$ (express in simplest form) (3)

(ii) $g(x)=(2x+1)^2e^{x^2}$ (do not simplify) (2)

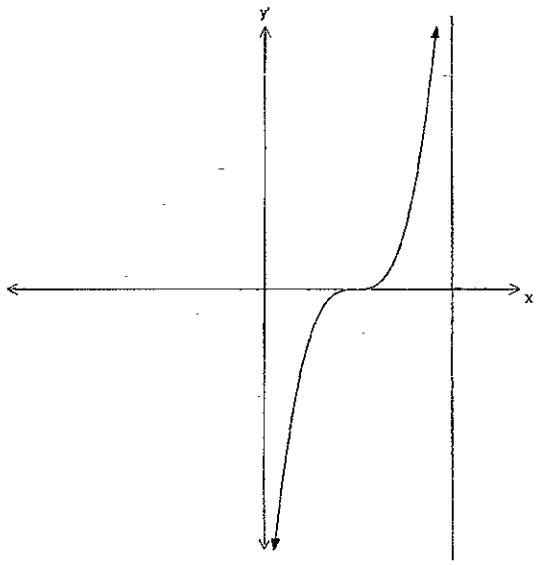
(b) For the graph already drawn, sketch the derivative function on the axes below. (2)

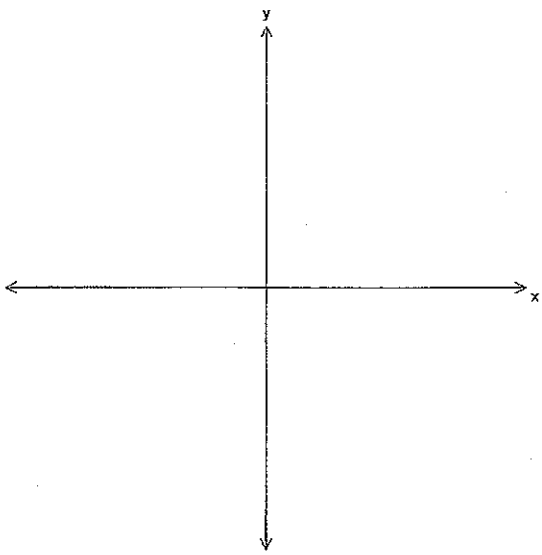




Question 2 (continued)

(c) Given the **derivative function**, sketch a possible graph of the function. (2)





Question 3 **(7 marks)**

(a) It is claimed that the tangent line to the curve $y = x^3 - 2x^2 - 4x + 3$ at $x=1$ passes through the point (3,8). Is this claim valid? Justify your answer. (5)

(b) Two identical biased coins are tossed together, and the outcome is recorded. After a large number of trials it is observed that the probability that both coins land showing heads is 0.36.

What is the probability that both coins land showing tails? (2)

Question 4 **(8 marks)**

The volume of a certain rectangular box is given by the equation $V = x^3 - 5x^2 - 8x + 48$

(i) The height of the box is $(4-x)$ units. (2)

Show that $\frac{x^3-5x^2-8x+48}{4-x}$ results in the expression $-x^2 + x + 12$.

(ii) In the context of this question, what does $-x^2 + x + 12$ represent? (1)

(iii) Calculate the value of x for which the volume is a maximum. (5)

Question 5 **(10 marks)**

(a) Find the global maximum and minimum values over the interval $\frac{1}{2} \leq x \leq 2$ (5)

of the function $y = x + \frac{1}{2x^2}$

(b) Events A and B are such $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(\overline{A \cup B}) = \frac{1}{4}$

(i) Show that event A and B are **NOT** mutually exclusive. (3)

(ii) Hence find $P(A \cap B)$. (2)

Question 6 **(8 marks)**

(a) Differentiate $y = \sqrt{3x^2 + 4}$ by letting $u = 3x^2 + 4$ and using the chain rule.

Show your working. (2)

(b) Determine $\int 3x^3(2x^4 - 5)^8 dx$ (2)

(c) $f(x)$ is defined such that $\int_{-3}^{6} f(x)\,dx = 24$ and $\int_{2}^{6} f(x)\,dx = 36$

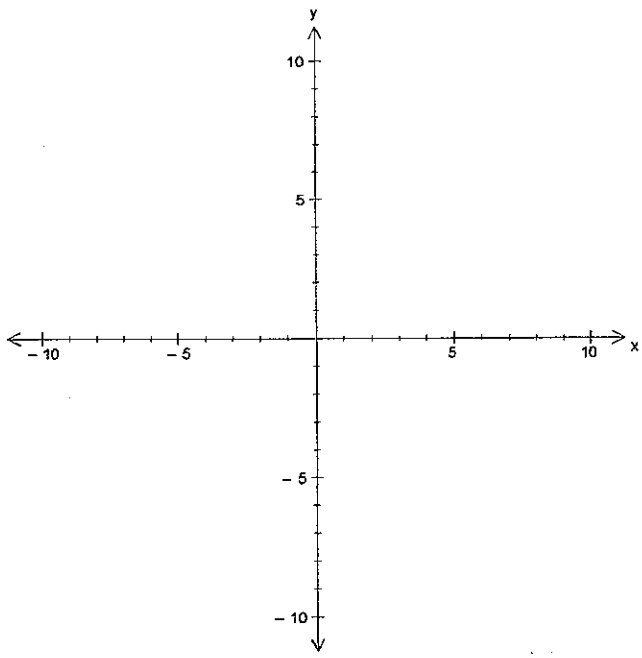
Find

(i) $\int_{-3}^{2} f(x)dx$. (1)

Question 6 (continued)

(c) (ii) $\int_{-3}^{2} \left(4f(x)+3\right) dx$ (2)

(iii) Sketch a possible graph of $y=f(x)$ for $-3 \leq x \leq 6$. Your graph should display the relative areas of important regions but you do not need to draw this graph to scale. (1)



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